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PPGEP Programa de Pós-Graduação em Engenharia de Produção
UTFPR UNIVERSIDADE TECNOLÓGICA FEDERAL DO PARANÁ
IA nas Engenharias
29 nov. a 01 de dezembro 2023

Portfolio Optimization with GARCH Models Using Multiple Time Windows for Pareto Frontiers

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Abstract: This study introduces a novel approach for portfolio optimization by employing Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to assess risk and construct Pareto frontiers over multiple time windows. Traditional risk measures such as the standard deviation may fail to capture risk accurately in financial markets where volatility is time-varying. By modeling conditional volatility, GARCH models offer a more comprehensive depiction of risk. In this paper, we implement this approach in the non-diversified portfolio optimization of two commodities, corn and soy, projecting their price dynamics 252 days ahead. The results demonstrate that the application of GARCH models and multi-period Pareto frontiers can significantly enhance portfolio optimization, providing fresh insights into purchasing and selling opportunities in the commodities market. This study adds to the current literature by addressing the gaps in applying Pareto frontiers across multiple time windows and utilizing GARCH models for risk measurement.

Keywords: MultiObjective Portfolio Optimization, GARCH Models, Pareto Frontiers.

Otimização de Portfólio com Modelos GARCH Usando Múltiplas Janelas Temporais para Frentes de Pareto

Resumo: Este estudo introduz uma abordagem inovadora para a otimização de portfólio, empregando Modelos de Heterocedasticidade Condicional Autoregressiva Generalizada (GARCH) para avaliar o risco e construir fronteiras de Pareto em múltiplas janelas de tempo. Medidas de risco tradicionais, como o desvio padrão, podem falhar em capturar o risco de forma precisa em mercados financeiros onde a volatilidade varia com o tempo. Ao modelar a volatilidade condicional, os modelos GARCH oferecem uma representação mais abrangente do risco. Neste artigo, implementamos essa abordagem na otimização de portfólio não diversificado de somente duas commodities, milho e soja, projetando suas dinâmicas de preço 252 dias à frente. Os resultados demonstram que a aplicação de modelos GARCH e fronteiras de Pareto de múltiplos períodos pode aprimorar significativamente a otimização de portfólio, fornecendo novos insights sobre oportunidades de compra e venda no mercado de commodities. Este estudo contribui para a literatura atual, abordando as lacunas na aplicação de fronteiras de Pareto em múltiplas janelas de tempo e utilizando modelos GARCH para a medição de risco.

Keywords: Otimização de Portfólio Multiobjetivo, Modelos GARCH, Fronteiras de Pareto.

1. Introduction

Portfolio optimization is a critical aspect of investment management, aiming to maximize return for a given level of risk. Since the seminal work of MARKOWITZ (1952), various models and approaches have been proposed to tackle this problem. However, traditional risk measures such as standard deviation may not adequately capture risk in financial markets, where volatility can change significantly over time.

This challenge becomes particularly salient in the context of commodities trading. Commodities, especially grains like corn and soy, are not only essential for food production but also have significant economic importance[1]. The markets for these commodities are characterized by their complex dynamics and high volatility, making the traditional portfolio optimization models potentially inadequate. These complexities demand innovative strategies and tools for risk measurement and portfolio optimization.

In light of these complexities, this study proposes a novel approach to portfolio optimization using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to measure risk and construct Pareto frontiers across multiple time windows. Our approach overcomes the limitations of traditional risk measures, offering a more realistic and effective strategy for portfolio optimization in commodity markets.

The use of GARCH models allows for more accurate capture of the temporal changes in volatility, providing a more nuanced understanding of risk and subsequently identifying optimal portfolios for trading commodities like corn and soy. Our approach can thus offer new insights into buying and selling opportunities in the commodities market, contributing to more informed decision-making and improved risk management strategies.

Moreover, our approach has profound implications for agricultural economies, global food security, and people's livelihoods worldwide, as fluctuations in commodity prices can significantly impact these areas. By providing more accurate and effective portfolio optimization strategies, our approach can help mitigate these impacts, promoting more efficient and sustainable commodity trading.

The rest of the paper is structured as follows: Section 2 provides a review of the relevant literature, highlighting the importance of portfolio optimization in commodity trading and the shortcomings of traditional approaches. Section 3 describes the methodology used in this study, including the use of GARCH models and multi-period Pareto frontiers. Section 4 presents the results and discussion, analyzing the findings and their implications. Finally, Section 5 concludes the paper, summarizing the key findings and suggesting directions for future research.

2. Portfolio Optimization and Risk Measurement: A Theoretical Overview

Our methodology initiates with an in-depth data exploration, examining price, return, and distribution trends over time. This crucial step, rooted in financial econometrics, assists us in understanding the behavior of the financial time series, identifying any potential patterns, anomalies, and distributional properties. This, in turn, guides the selection of suitable statistical models like GARCH that capture time-varying volatility effectively. The ultimate objective is to adapt our modeling approach based on data characteristics, which helps in improving the accuracy of GARCH model estimations and portfolio optimization.

[1]According to recent data from the Food and Agriculture Organization (FAO), global grain production reached a record high of 2.75 billion metric tons in 2022, highlighting the importance of efficient commodity trading and risk management strategies.

2.1 GARCH Model Estimation

The estimation results of various GARCH models, including GARCH(1,1)std, sGARCH(1,1)sstd, EGARCH(1,1), GJRGARCH(1,1), and APARCH(1,1), were compared based on several criteria. The chosen model, sGARCH(1,1)sstd, exhibited favorable characteristics such as a lower number of estimated parameters, a competitive minimum squared error, a higher likelihood, and relatively low information criteria[2].

The sGARCH(1,1)sstd model can be mathematically represented as:

$$\sigma_t^2 = \omega + \alpha \cdot \epsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2 + \gamma \cdot (\epsilon_{t-1}^2 - \sigma_{t-1}^2)$$

where σ_t^2 is the conditional variance at time *t*, ϵ_{t-1}^2 is the squared residual at time $t-1$; ω is the intercept term; α is the autoregressive coefficient for the ARCH effect; β is the autoregressive coefficient for the GARCH effect; γ is the autoregressive coefficient for the leverage effect.In this model, the conditional variance at time *t*, σ_t^2 is a weighted combination of the squared residual at time *t-1*, ϵ_{t-1}^2, and the conditional variance at time *t-1*, σ_{t-1}^2.The coefficient α determines the impact of past squared σ_{t-1}^2 residuals on the current variance, while the coefficient β captures the persistence of the past conditional variance. The coefficient γ represents the leverage effect, which measures the asymmetry in the response of the conditional variance to positive and negative shocks.

The estimation of the parameters, ω, α, β, and γ is typically done using maximum likelihood estimation. The sGARCH(1,1)sst model is widely used in financial time series analysis to capture the dynamics of volatility, including the asymmetric response to shocks.

2.2 Portfolio Optimization and Multiple Pareto Frontiers

We utilized the estimated parameters of the sGARCH(1,1)sstd model obtained from the in-sample data spanning from January 1, 2018, to July 1, 2023, to simulate the trajectory of returns, prices, and volatilities for the next 252 future trading days. This simulation forms the basis for conducting portfolio optimization calculations and identifying windows of opportunity for buying and selling within this simplified portfolio comprising only two assets, which entails limited diversification. By leveraging these results, we can assess the potential performance of the portfolio under different market scenarios and make informed investment decisions. It is worth noting that while this simplified portfolio may lack diversification, our focus lies on evaluating the effectiveness of the sGARCH(1,1)sstd model and exploring potential opportunities within this constrained context.

The Pareto frontier was obtained based on the traditional portfolio optimization problem, formulated as:

$$\max \sum_{i=1,t}^{n} \mu_i w_{i,t}, \ \min \sum_{i=1,t}^{n} \sum_{j=1,t}^{n} \sigma_{ij} w_{i,t} w_{j,t}$$

[2] See Results section to see these findings.

Subject to:

$$\sum_{i=1,t}^{n} w_{i,t} \geq 0,\ \min \sum_{i=1,t}^{n} \mu_{i,t} \geq \text{int. rate},\ \sigma^2(w)_{i,t} \leq \sigma^2_{max}$$

Considering the weights for only two commodities portfolio, obtained by

$$w^*_{1,t} = \frac{\sigma^2_{2,t} - \sigma_{12,t}}{\sigma^2_{1,t} + \sigma^2_{2,t} - 2\sigma_{12,t}}$$

where $w_{i,t}$ represents the asset weights for each commodity *i* at time *t*, $\mu_{i,t}$ denotes the expected returns of the assets, $\sigma_{ij,t}$ indicates the covariance between assets *i* and *j* at the range of time *t*, μ_{min} denotes the minimum acceptable return, and σ_{max} represents the maximum acceptable risk and the constraint $\mu_i \geq$ needs to be equal or higher than the actual interest rate (FED, interest rates, preferentially).

By incorporating the Differential Evolution algorithm (for more details about see ARDIA et al., 2011) available in R language, into the portfolio optimization process, we aim to explore alternative paths for achieving Pareto fronts across multiple time windows, enhancing the robustness and efficiency of the optimization results.

We construct the Pareto frontiers for multiple time windows to identify the optimal portfolios. The Pareto frontier represents the set of portfolios that offer the highest expected return for each level of risk. By examining how the optimal portfolios change over time through the construction of Pareto frontiers for multiple time windows, we gain insights into the dynamics of optimal portfolio compositions.

The portfolio optimization problem is solved using a numerical optimization algorithm, and the results are analyzed to identify buying and selling opportunities in the commodities market.

This methodology allows us to address the research objectives by providing a more accurate measure of risk and identifying the optimal portfolios over multiple time windows.

3. Portfolio Optimization and Risk Measurement: A Theoretical Overview

Markowitz´s (1952) portfolio optimization model is a cornerstone approach to risk management in investments, aiming to maximize return for a given level of risk. The standard risk measure, the standard deviation, is a measure of dispersion that considers the variability of returns relative to the mean over a given time period. However, this measure may not adequately capture risk in certain situations, especially when the volatility of returns changes over time. In particular, standard deviation does not account for the asymmetry in volatility increases and decreases, a phenomenon often observed in financial markets.

GARCH models, proposed by Engle (1982) and Bollerslev (1986), are able to model conditional volatility, which means they can capture changes in volatility over time. This is particularly useful in financial markets, where volatility can change significantly from one day to the next. By modeling conditional volatility, GARCH models can provide a more accurate measure of risk, which can be crucial for effective portfolio optimization. Importantly, GARCH models allow for the modeling of leverage effects, where volatility increases are not symmetric with volatility decreases.

A systematic literature review was conducted to identify recent studies that address portfolio optimization and the application of Pareto fronts or efficient frontiers across different time windows. Despite the extensive literature on portfolio optimization, it was observed that there is a scarcity of studies that explicitly apply Pareto fronts or efficient frontiers across multiple time windows, considering too the aspect of the appropriate dynamic risk measures. This suggests a gap in the current literature and indicates the need for further research in this area to better understand how efficient frontiers may vary over time and how this variation can be used to enhance portfolio optimization.

Lu et al. (2021) present a multiobjective and multiperiod portfolio selection model that addresses uncertainty in asset returns. The study aims to optimize resource allocation in a portfolio by considering three key criteria: mean, semientropy, and skewness. Mean represents the expected return, semientropy accounts for the risk associated with uncertain returns, and skewness measures the distribution's asymmetry. The authors incorporate these objectives into the model, focusing on maximizing mean, minimizing semientropy, and minimizing skewness. They utilize fuzzy set theory and employ the NSGA-II algorithm for optimization, while considering various realistic constraints. Additionally, the authors evaluate the performance of the proposed model by analyzing the Pareto fronts for different time horizons. The main finding is that the model effectively manages uncertainty and achieves efficient portfolio allocations by incorporating mean, semientropy, and skewness as optimization criteria.

By other hand, Zyl et al. (2022) argues that in practice, portfolio management is often treated as a single-period problem due to the computational challenges of constructing a multi-period Pareto frontier. The authors present a framework called Pareto Driven Surrogate (ParDen-Sur) to efficiently perform the required hyper-parameter search. The authors evaluate this framework against, and in conjunction with, several seminal Multi-Objective (MO) EAs on two datasets for both the single-and multi-period use cases. The results show that ParDen-Sur can speed up the exploration for optimal hyper-parameters by almost 2× with a statistically significant improvement of the Pareto frontiers, across multiple EAs, for both datasets and use cases.

The authors also discuss the traditional approaches to Multi-period Portfolio Optimisation (MPO) and its associated challenges. They highlight the advantages of taking a multi-period perspective of the portfolio optimization problem, such as management of trading costs, time-varying forecasts, intertemporal hedging, and other intertemporal constraints. However, they also note that MPO is computationally burdensome, especially when incorporating real-world considerations.

Finally, Gupta et al. (2020) propose a multi-period portfolio selection model under intuitionistic fuzzy conditions. This model, aimed at maximizing terminal wealth and minimizing cumulative risk, incorporates practical constraints such as full capital utilization, prohibition on short selling, and static transaction costs, among others. Their model innovatively factors in hesitation in decision-making, thereby enhancing portfolio performance. The model introduces parameters θW and θVa to represent hesitation, leading to two categories: optimistic and pessimistic portfolio selection models for respective investor types. The proposed models are solved using a max-min approach.

In summary, although there have been notable advancements in portfolio optimization, specific gaps still exist in the literature, specifically regarding utilizing Pareto fronts across multiple time windows and incorporating GARCH models for risk assessment.

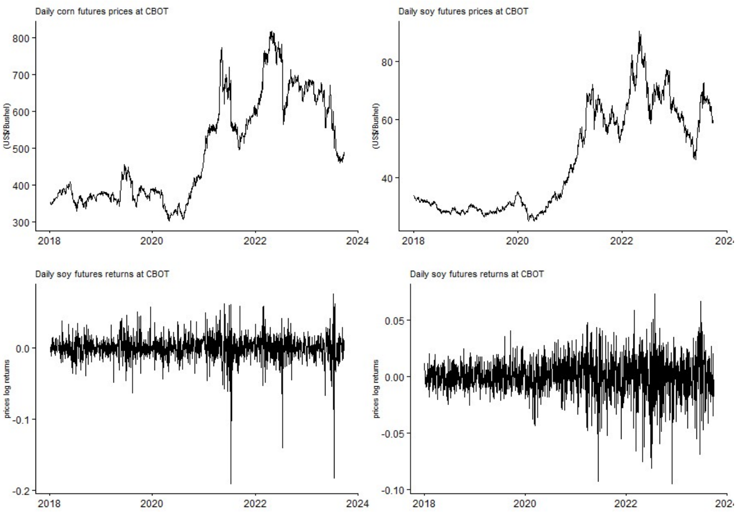
This study aims to fill these gaps by introducing a novel approach that involves employing GARCH models to measure risk and constructing Pareto fronts across multiple time windows. By exploring this alternative path, we seek to provide valuable insights and opportunities for analysis in the field of portfolio optimization.

4. Results and discussion

The results discussion section provides an analysis of the key findings obtained in this portfolio optimization study using GARCH models and multiple time window Pareto frontiers. In this section, we delve into the results obtained from simulating the trajectory of returns, prices, and volatilities of assets for the next 252 trading days. The data for time series quotes are available at the Yahoo!Finances API. Furthermore, we explore the implications of these findings in terms of portfolio performance and identify potential investment opportunities. The discussion revolves around the application of the GARCH model for risk measurement and the construction of Pareto frontiers across different time windows.

We commence by assessing the temporal patterns of corn and soybean prices, as well as their corresponding returns, throughout the analyzed period. This initial step allows us to gain insights into the dynamics and fluctuations exhibited by these agricultural commodities, setting the foundation for our subsequent analysis.

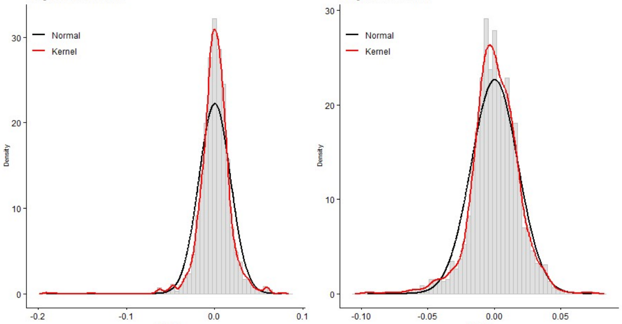
Figure 1 – Daily Corn and Soy CBOT time series for prices and returns



Source: Yahoo!Finances API (2023)

Continuing along the same line of reasoning, it is crucial to evaluate the distributions of individual asset returns. By examining the distributional characteristics of corn and soybean returns separately, we gain a deeper understanding of the underlying volatility and risk associated with each asset. This analysis allows us to identify potential asymmetries, fat tails, or other noteworthy features in the return distributions that may impact portfolio performance. Furthermore, assessing the individual asset distributions provides insights into the behavior and dynamics specific to corn and soybean markets, enabling us to make informed decisions when constructing and optimizing the portfolio.

Figure 2 – Normal x Kernel Density plots for prices log-returns



Source: Devised by the authors

The histograms clearly depict that the distribution of returns deviates from a normal distribution, indicating the presence of non-normal behavior. It is noteworthy that the observed return series exhibit a higher probability of extreme negative returns compared to positive returns. This asymmetry in the return distribution is crucial for understanding the risk profile of the assets under investigation. Additionally, the kernel density curves provide a robust representation of the underlying distribution, capturing the heavy-tailed nature and providing valuable insights into the true pattern of the data. Overall, these findings highlight the importance of considering non-normal distributions and their implications for portfolio analysis and risk management.

Table 1: Normality hypothesis and flattening measures for commodities returns

Series	JB-Statistic	p-value	Skewness	Kurtosis	Type	Normality
Corn returns	10893.37	0.00	1.52	17.86	Leptocurtic	Not Normal
Soy returns	266.17	0.00	0.53	5.12	Leptocurtic	Not Normal

Source: Devised by the authors

The Jarque-Bera test (JB test) is a statistical test used to assess the normality assumption of a dataset. It examines the skewness and kurtosis of the sample to determine if the data follows a normal distribution. In our analysis, the JB test is relevant due to the observed non-normal behavior of the return distributions, which exhibit leptokurtic characteristics. Leptokurtic distributions have higher kurtosis and heavier tails, indicating a greater likelihood of extreme values. Recognizing the leptokurtic nature of the data is essential for accurate risk assessment and informed investment decision-making.

By running the family of GARCH models tested, we need to select the best one who makes sense with this kind of returns distributional patterns and with the market behavior. We tryed the following models specifications showed in the table below and we used the following metrics criteria to select the best one:

Table 2: Params count, likelihood and MSE for model selection criteria

Model	Params	Likelihood	MSE
$\log(\mathrm{Corn}_t / \mathrm{Corn}_{t-1})$ APARCH(1,1)	6	3067.81	1.771200e-06
$\log(\mathrm{Corn}_t / \mathrm{Corn}_{t-1})$ EGARCH(1,1)	5	3063.46	1.785472e-06
$\log(\mathrm{Corn}_t / \mathrm{Corn}_{t-1})$ GARCH(1,1)std	5	3111.56	1.849858e-06
$\log(\mathrm{Corn}_t / \mathrm{Corn}_{t-1})$ GJRGARCH(1,1)	5	3059.07	1.868688e-06
$\log(\mathrm{Corn}_t / \mathrm{Corn}_{t-1})$ sGARCH(1,1)sstd	8	3112.02	1.843572e-06
$\log(\mathrm{Soy}_t / \mathrm{Soy}_{t-1})$ APARCH(1,1)	6	2994.52	4.926742e-07
$\log(\mathrm{Soy}_t / \mathrm{Soy}_{t-1})$ EGARCH(1,1)	5	2991.10	4.920431e-07
$\log(\mathrm{Soy}_t / \mathrm{Soy}_{t-1})$ GARCH(1,1)std	5	2998.79	4.961854e-07
$\log(\mathrm{Soy}_t / \mathrm{Soy}_{t-1})$ GJRGARCH(1,1)	5	2989.61	4.945409e-07
$\log(\mathrm{Soy}_t / \mathrm{Soy}_{t-1})$ sGARCH(1,1)sstd	8	3005.06	4.666231e-07

Source: Devised by the authors

And by using information criteria, we can see:

Table 3: Model selection by Information Criteria

Model	Akaike	Bayes	Shibata	Hannan-Quinn
$\log(\mathrm{Corn}_t / \mathrm{Corn}_{t-1})$ GARCH11std	-5.46	-5.44	-5.46	-5.45
$\log(\mathrm{Soy}_t / \mathrm{Soy}_{t-1})$ GARCH11std	-5.26	-5.24	-5.26	-5.25
$\log(\mathrm{Corn}_t / \mathrm{Corn}_{t-1})$ sGARCH11sstd	-5.46	-5.42	-5.46	-5.44

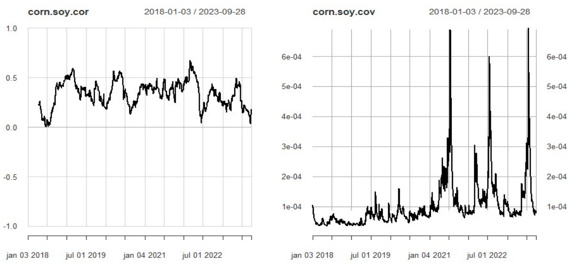
$\log(\text{Soy}_t / \text{Soy}_{t-1})$ sGARCH11sstd	-5.27	-5.23	-5.27	-5.25
$\log(\text{Corn}_t / \text{Corn}_{t-1})$ EGARCH11	-5.38	-5.35	-5.38	-5.37
$\log(\text{Soy}_t / \text{Soy}_{t-1})$ EGARCH11	-5.25	-5.23	-5.25	-5.24
$\log(\text{Corn}_t / \text{Corn}_{t-1})$ GJRGARCH11	-5.37	-5.35	-5.37	-5.36
$\log(\text{Soy}_t / \text{Soy}_{t-1})$ GJRGARCH11	-5.25	-5.22	-5.25	-5.24
$\log(\text{Corn}_t / \text{Corn}_{t-1})$ APARCH11	-5.38	-5.36	-5.38	-5.37
$\log(\text{Soy}_t / \text{Soy}_{t-1})$ APARCH11	-5.25	-5.23	-5.25	-5.25

Source: Devised by the authors

In light of the findings, the sGARCH(1,1)sstd model offers a compelling balance between complexity and performance. Despite having more parameters than the GARCH(1,1)std model, which could potentially lead to overfitting, the sGARCH11sstd model demonstrates higher likelihoods, indicating a better fit to the data. The leptokurtic nature of financial return data necessitates a model that can adequately represent these characteristics, a requirement fulfilled by the skewed Student's *t*-distribution incorporated in the sGARCH11sstd model. Furthermore, this model's lower Mean Squared Error for soy returns suggests a more accurate forecasting ability. While the sGARCH(1,1)sstd model does not hold the lowest information criteria, its overall performance across different return data, its robustness and its superior forecasting accuracy for soy returns make it the chosen model for forecasting/scenario simulation.

Based on the selected GARCH model covariances in the sample, we can plot the 3 months correlation between the two commodities returns along the time and in the same way we can inspect the covariances:

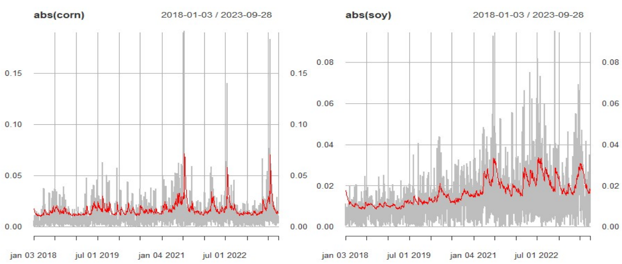
Figure 3 – 3-months rolling correlations between corn and soy returns and covariance pattern



Source: Devised by the authors

And we can see the in-the-sample sGARCH(1,1)sst estimated volatilities:

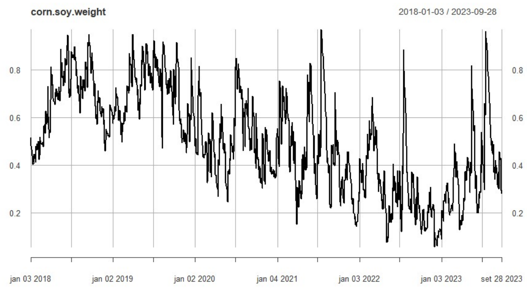
Figure 4 – In the sample sGARCH(1,1)sst daily returns corn and soy volatilies



Source: Devised by the authors

The sGARCH(1,1)sst model enables us to estimate the dynamic weights of the daily portfolio combination using the returns and volatilities of soybean and corn. These weights are derived from the model's estimated conditional variances and covariances.

Figure 5 - Corn and Soy portfolio dynamic weights along time period



Source: Devised by the authors

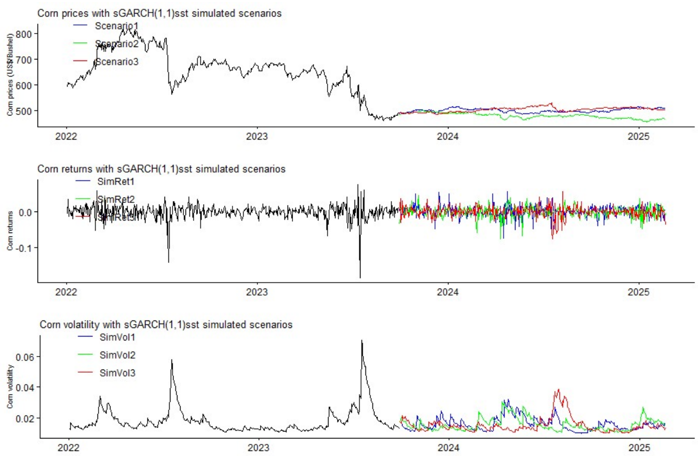
The portfolio strategy exhibits clear dynamism, with the optimal weight of corn and soybean commodities varying between 10 and 90% throughout the period based on the values of variances and covariances.

By incorporating the time-varying volatilities and correlations, the sGARCH(1,1)sst model allows for adaptive portfolio allocation that responds to changing market conditions. This dynamic approach enhances risk management and performance by adjusting the portfolio weights based on evolving risk dynamics in the soybean and corn markets.

Estimating daily volatilities using the sGARCH(1,1)sst models provides a more accurate representation of the day-to-day variability in soybean and corn returns compared to using standard deviations or other measures. The GARCH models capture both the conditional mean and conditional variance, allowing for time-varying volatilities.

This is important in agricultural markets, where prices can experience significant fluctuations on a daily basis, in special peaks motivated by external events outside the traditional balance between supply and demand, as Ukraine war begins and Covid pandemic. By considering the estimated volatilities from the GARCH models, we gain valuable insights into the risk and variability present in these markets, enabling better portfolio management and decision-making. The sGARCH(1,1)sst models, with their incorporation of leverage and asymmetry effects, further enhance their ability to capture the unique characteristics of agricultural commodity returns.

Figure 6 - Simulated Scenarios for Corn prices, returns and volatilities with sGARCH(1,1)sst

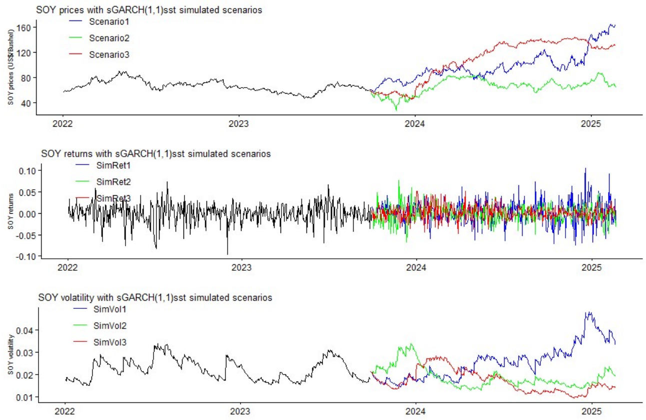


Source: Devised by the authors

We can use sGARCH(1,1)sst simulated path for 3 different scenarios for the next 252 trading days in the future (in red, green and blue in the graph above) and use this forecasted scenario to make the Pareto frontiers for each quarter (22 days x 3 months). The strategic utilization of Pareto frontiers across the subsequent four quarterly timeframes (4 quarters, 22 days and 3 months) holds significant promise for decision-makers, as it offers a robust framework for identifying and capitalizing on forthcoming windows of opportunity.

After estimating the forecastings of returns for the next 252 days in the three scenarios, we utilized the arithmetic means of the three scenarios to construct the Pareto frontiers for each future quarter.

Figure 6 - Simulated Scenarios for Soy prices, returns and volatilities with sGARCH(1,1)sst



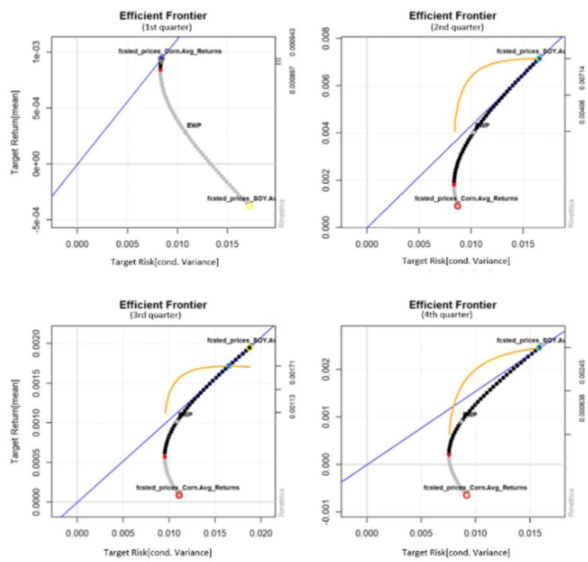
Source: Devised by the authors

We can use sGARCH(1,1)sst simulated path for 3 different scenarios for the next 252 trading days in the future (in red, green and blue in the graph above) and use this forecasted scenario to make the Pareto frontiers for each quarter (22 days x 3 months).

The strategic utilization of Pareto frontiers across the subsequent four quarterly timeframes (4 quarters, 22 days and 3 months) holds significant promise for decision-makers, as it offers a robust framework for identifying and capitalizing on forthcoming windows of opportunity.

After estimating the forecastings of returns for the next 252 days in the three scenarios, we utilized the arithmetic means of the three scenarios to construct the Pareto frontiers for each future quarter.

Figure 7 - Pareto Front for different quarters periods for Corn and Soy portfolio



Source: Devised by the authors

In the projected first quarter of the future, the analysis of the average scenario reveals a preference for Corn positions over Soy due to a better risk-return combination, changing in the second quarter, the Soy positions emerge as the preferred choice over Corn, as indicated by the appearance of the yellow line representing the Capital Market Line (CML).

The CML represents the efficient combination of risk and return based on the market portfolio and risk-free asset. The shift in preference towards Soy reflects a favorable risk-return tradeoff and potential market dynamics specific to that period.

Although Soy prevails in the projected preferences over Corn in the simulated scenario within the Pareto frontiers, this exercise demonstrates the significance of constructing Pareto frontiers for each temporal period to align with the periodic strategies of individual portfolio holders. By incorporating time-specific factors and investor objectives, such as changing market conditions and risk preferences, the tailored Pareto frontiers provide valuable insights and guide investment decisions. This approach ensures that portfolio allocations are contextually relevant, enabling investors to adapt and optimize their strategies based on the evolving dynamics of the market and their specific investment goals.

This highlights the importance of continuously monitoring and adapting portfolio allocations to capitalize on evolving market conditions and optimize investment performance. As an expert, recognizing these changes in preferences and adjusting portfolio composition accordingly can enhance risk management and potentially lead to better investment outcomes.

Changes in the composition of Pareto frontiers on a quarterly basis are influenced by market conditions, economic indicators, seasonality, investor behavior, and financial innovation. These factors impact risk-return profiles, market trends, and investor preferences, leading to shifts in optimal portfolio compositions.

By analyzing these graphs, investors can assess the trade-off between risk and return and select portfolios that align with their risk preferences. The Efficient Frontier allows investors to identify portfolios that offer the highest expected returns given their desired level of risk. The different graphs for each quarter provide insights into how the Efficient Frontier may change over time as market conditions and expectations evolve.

5 Conclusion and Future Research Directions

In this study, we embarked on an investigation into the optimal portfolio allocation for agricultural commodities, namely corn and soybean, using GARCH models and multiple time window Pareto frontiers. Our objective was to identify the best portfolio strategy that maximizes returns while minimizing risk.

We utilized GARCH family models due to their ability to capture the non-normal and leptokurtic nature of commodity return distributions, which is a crucial aspect of portfolio risk assessment. By incorporating time-varying volatilities and correlations into our portfolio analysis, we were able to devise a dynamic portfolio allocation strategy that is adaptive to the changing risk-return trade-off in the corn and soybean markets.

Our study revealed significant variation in the daily prices, returns, and volatilities of corn and soybean commodities, with clear evidence of leptokurtosis and heavy tails in the return distributions. This points to the importance of utilizing GARCH models in portfolio optimization, as they can accommodate the unique characteristics of commodity return distributions and provide more accurate risk estimates.

However, it's important to recognize the limitations of our approach. While the sGARCH(1,1)sst model exhibits robust forecasting ability, it may fail to capture certain uncertainties inherent in economic time series. These can include structural breaks caused by exogenous events, the impact of successive and imminent crises, and random events that fall outside the purview of traditional econometric price formation models. Thus, the portfolio strategy derived from this model should be employed with an understanding of these potential shortcomings.

The application of multiple time window Pareto frontiers allowed us to view the evolution of the optimal portfolio over different time horizons, providing a more nuanced understanding of the optimal trade-off between risk and return over time.

Our study, therefore, contributes to the ongoing discourse on portfolio optimization in agricultural commodity markets by offering a robust, dynamic, and flexible portfolio strategy that can adapt to the evolving risk-return landscape in these markets. This approach can be beneficial for a range of market participants, including farmers, commodity traders, and investment managers.

Future research can expand on our work by incorporating other agricultural commodities into the portfolio, examining other types of GARCH models, or considering different portfolio optimization criteria. Additionally, exploring the impact of other economic factors and incorporating them into the GARCH model could further enhance our understanding of the dynamics in agricultural commodity markets.

In conclusion, portfolio optimization using GARCH models and multiple time window Pareto frontiers offers a compelling approach to managing risk and maximizing returns in agricultural commodity markets. Through our study, we highlighted the value of this approach in the context of corn and soybean commodities, providing insights that can inform effective portfolio management strategies in these markets.

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