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Geometric Aspects to Perform Maneuver in the Automatic Parking System

Calequela João Tomé Manuel[1]

PhD student of PPGEP - Federal Technological University of Paraná, Ponta Grossa, PR, Brazil

Giane G. Lenzi[2]

Production Engineering - Federal Technological University of Paraná, Ponta Grossa, PR, Brazil

Max M. D. Santos[3]

Production Engineering - Federal Technological University of Paraná, Ponta Grossa, PR, Brazil

Angelo M. Tusset[4]

Production Engineering - Federal Technological University of Paraná, Ponta Grossa, PR, Brazil

Abstract: This paper presents the planning and geometric path used to perform the automatic parking maneuver in parallel form, as well as the main variables used in the domain of maneuver constantly required in the algorithms of automatic parking of vehicles; in order to ensure greater security and less parking time.

Keywords: Automatic parking, Pythagorean theorem, Geometric modeling, Ackerman steering.

Aspectos Geométricos para Realizar Manobras em Sistema de Estacionamento Automático

Resumo: Este artigo apresenta o planejamento e caminho geométrico utilizados para executar a manobra do estacionamento automático de veículos na forma paralela, assim como as principais variáveis utilizadas no domínio da manobra requisitados constantemente nos algoritmos de estacionamento automático de veículos; a fim de garantir maior segurança e menor tempo de estacionamento.

Palavras-chave: Estacionamento automático, Teorema de Pitágoras, Modelagem geométrica, Direção de Ackerman.

1. Introduction

Road vehicle is a machine that transport peoples and loads from start point to destination point (GILLESPIE, 1992). It is a dynamic system that maneuvers should be carried out to get the proper motion on the ground in safety and comfort. Kinematics is part of physics or mechanics that studies the movement of bodies in relation to their direction, distance, meaning, orientation and dynamics is part of physics or mechanics that studies the motion of bodies, with respect to forces, velocity, acceleration, mass acting on it (MANUEL et al., 2021).

In the past, for most vehicles to perform a maneuver, the driver had to change gears mechanically and use the rearview mirror as a guide without any help or technological intervention unless it was a vehicle equipped with automatic or automated transmission to facilitate maneuvering (CALLEJON, 2017). Remember the driver's double attention when maneuvering to avoid colliding with other vehicles or objects (ORLOVSKA et al., 2020). To facilitate parallel or perpendicular parking maneuvers, some companies have developed systems that can assist the driver in this type of maneuver without causing harm to others (WU et al., 2016). This type of parking control consisted of sensors that detected the distance the vehicle was in another vehicle or objects (GUPTA; DIVEKAR, 2010) and to obtain such a system in a common vehicle or even a simple reverse sensor, the solution was buy a vehicle with option of package or a luxury template (SACHIN et al., 2019)

The geometric representation of a parallel automatic parking system (MANUEL et al, 2020) is valid when it aggregates physical/mechanical characteristics of kinematic (WRIGHT, 1898) that directly influence the physical/mechanical characteristics of dynamic (DRESIG; HOLZWEIßIG, 2010) to ensure greater precision during its applicability. Table 1 describes the main relevant characteristics of kinematics and dynamics.

Table 1 – relevant characteristics of kinematics and dynamics

Kinematics	Dynamics
• Calculates the trajectory of bodies or particles.	➢ Calculates the movement and causes of the movement.
• It does not take into account the amount of mass of each body or particle in the system.	➢ It takes into account the mass and forces of each particle in the system.
• It can be considered a branch of mathematics.	➢ It is a branch of physics that cannot be regarded as a branch of mathematics.

Source: Adapted from MANUEL et al (2020)

Research on geometric aspects for maneuvers used in parallel automatic parking systems has been highlighted in recent years. Being the basis for the development of an automatic parking algorithm, considering some physical and mechanical dimensions of the vehicles and the parking space (width and length). The bibliometric update of this paper was influenced by the articles highlighted in Table 2, in chronological order of years (from 2010 to 2020).

Table 2 – relevant characteristics of kinematics and dynamics

References	Objectives and Metrics	Final Considerations
(GUPTA, ANKIT.; DIVEKAR, ROHAN.; AGRAWAL, MOHIT., 2010).	This article explains in detail a simple and accurate parking algorithm for automatic parallel parking based on the Ackerman driving configuration.	The System uses sonar sensors and wheel encoders for its perception, the algorithm uses simple geometry for its path planning and odometry.

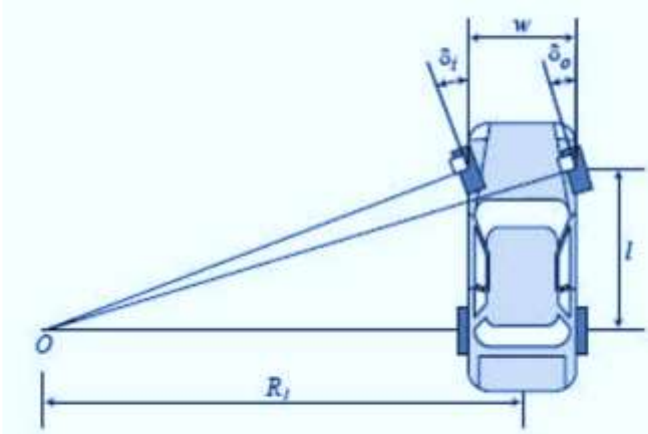
(RAZINKOVA, ANASTASIA.; CHO, HYUN-CHAN.; JEON, HONG-TAE., 2012).	In this work, an intelligent autoparking system is introduced that automatically generates the trajectory for parking using fuzzy logic. In the first part, we present the trajectory generation method for parallel parking without collisions. Trajectory based on fuzzy logic the generation algorithm is described in the second part.	The proposed intelligent trajectory generation algorithm has several advantages such as high robustness to the variance in parking parameters and the fact that the algorithm takes into account the velocity of vehicle.
(WU, TER-FENG.; TSAI, PU-SHENG.; HU, NIEN-TSU.; CHEN, JEN-YANG., 2016).	In this study, an ultrasonic sensor is placed in a mobile robot with smart wheels (SWMR) in order to detect the environment of the parking space on the side of the road and reverse.	After the ultrasonic sensors detect sufficient parking space, the SWMR can automatically control the wheel servomotors to turn and go in a straight line, move back and stop. According to experimental results, this study can simulate real parking situations.
(SACHIN, MR.; PATIL, S.; NATU, MR.; PATIL, KIRAN.; JAMDADE, MR., 2019).	The aim of this work is to propose an intelligent system that may be able to display the empty space on the back of the panel. This will help the driver to assess the rear empty space for parking using the ultrasonic sensor, Arduino, LCD 16 X 2 (Display).	We were able to successfully implement this system and thus we can replace the traditional parking system by the intelligent vehicle parking system. The panel system will provide information on the distance back to the driver. This system also helps the driver in reverse parking.
(ORLOVSKA, JULIA.; NOVAKAZI, FJOLLE.; BLIGARD, LARS-OLA.; KARLSSON, MARI ANNE.; WICKMAN, CASPER.; SODERBERG, RIKARD., 2020).	This article aims to investigate and understand how the driving context affects the use of ADAS. Data from a Naturalistic Driving (ND) study were collected and analyzed.	The results clearly indicate the triple interrelationship that includes driver behavior, system performance and driving context. driving context in the design and development of the system.

Source: Adapted from MANUEL et al (2020)

2. Geometrical modeling to the parking system

The automatic parking system is an available feature on the autonomous vehicle that allows carrying outmaneuvers automatically in order to move a vehicle from a traffic lane into a parking spot to perform parallel (GUPTA; DIVEKAR, 2010), perpendicular (MA et al., 2017) or oblique parking. First of all, the mathematical model of the parking maneuver should be described so that we can understand, analyze and design the controller and strategy to control vehicles along with the parking system. The differential between the two front wheels as show in Figure 1, because of the wheel steering angles (δ_i , δ_0) can be obtained from a trapezoidal tie rod arrangement (GILLESPIE, 1992).

Figure 1 – The differential between the two front wheels



Source: Adapted from JAZAR (2017)

However, the steering angle of the front wheels (δ_i , δ_0) is based on the characteristic of the Ackerman geometry (RAHMAN et al., 2021) represented by Equation 1 that determines the solution of this difference in position between the wheels.

$$cot\delta_0 - cot\delta_i = \frac{w}{l} \quad (1)$$

Where (w) is the distance between wheels on the same axle, and (l) is Length between axes. The external steering angle (δ_0) is determined by Equation 2.

$$\delta_o = tan^{-1}\frac{l}{R_1 + \frac{w}{2}} \quad (2)$$

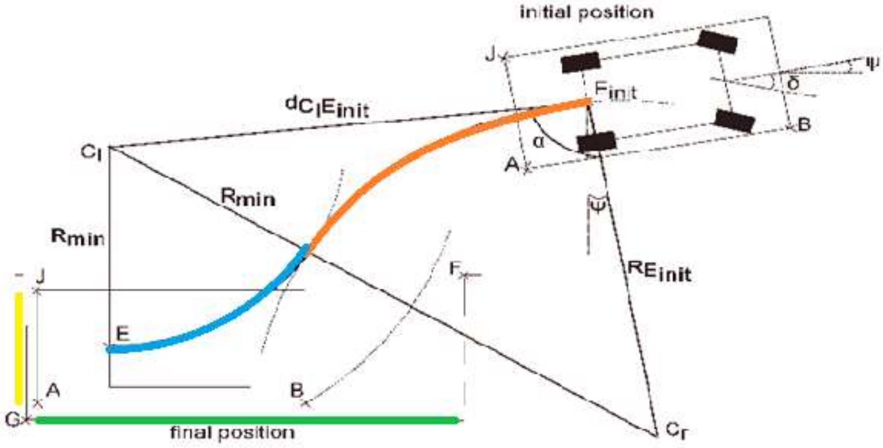
(R_1) is the distance from center of rotation (o) to the center/half of the distance (w). The inner steering angle (δ_i) is determined by Equation 3.

$$\delta_i = tan^{-1}\frac{l}{R_1 - \frac{w}{2}} \quad (3)$$

The maneuvering side (right or left) of your wheel always has a larger internal steering angle than the external. In this case, the wheel steering is directed to the right ($\delta_i > \delta_0$), its slip angle (β) is considered greater than zero (0), considering the positive counterclockwise direction. If the wheel steering is left ($\delta_0 > \delta_i$), its sliding angle (β) is considered less than zero (0), considering the negative clockwise direction (MANUEL et al., 2021.

And in case there is no steering $(\delta_i = \delta_0)$ and its slip angle (β) is considered zero (0). In Figure 2, the simplified lateral reference path kinematic model (HASSAN et al., 2021) is proposed, so that it is possible to start the vehicle trajectory by defining the starting point (arc circle orange), intermediate point (connection between the two circle arcs) and the end point (blue circle arc).

Figure 2 – Simplified lateral reference trajectory parallel



Source: Adapted from VOROBIEVA et al (2015)

Point J to A (yellow line) defines the minimum width of the vehicle parking space and point A to F (yellow line) defines the minimum length of the vehicle parking space.

The two circular arcs, both connected by a tangential point, as follows: The first arc projected counterclockwise (right) from point $\boldsymbol{E_{init}}$ to point $(\boldsymbol{C_l}, \boldsymbol{R_{E_{lmin}}})$ the second arc projected clockwise (left) part of the point $(\boldsymbol{C_r}, \boldsymbol{R_{E_{init}}})$ to the end point $\boldsymbol{E}$.

The nomenclatures of geometric variables presented in Figure 2 are described in Table 3.

Table 3 – nomenclatures of geometric variables

Variable	Nomenclature
ψ	Vehicle orientation angle
δ	Wheel steering angle
E_{init}	Center of rear axle
α	Angle opposite distance $R_{E_{lmin}} + R_{E_{init}}$
$R_{E_{init}}$	Distance from point E_{init} to point C_r
$d_{C_l E_{init}}$	Distance from point E_{init} to point C_l
$R_{E_{lmin}} + R_{E_{init}}$	Distance from point C_r to point C_l
$E_{init}\ C_r\ C_l$	Triangle ($E_{init}\ C_r\ C_l$)

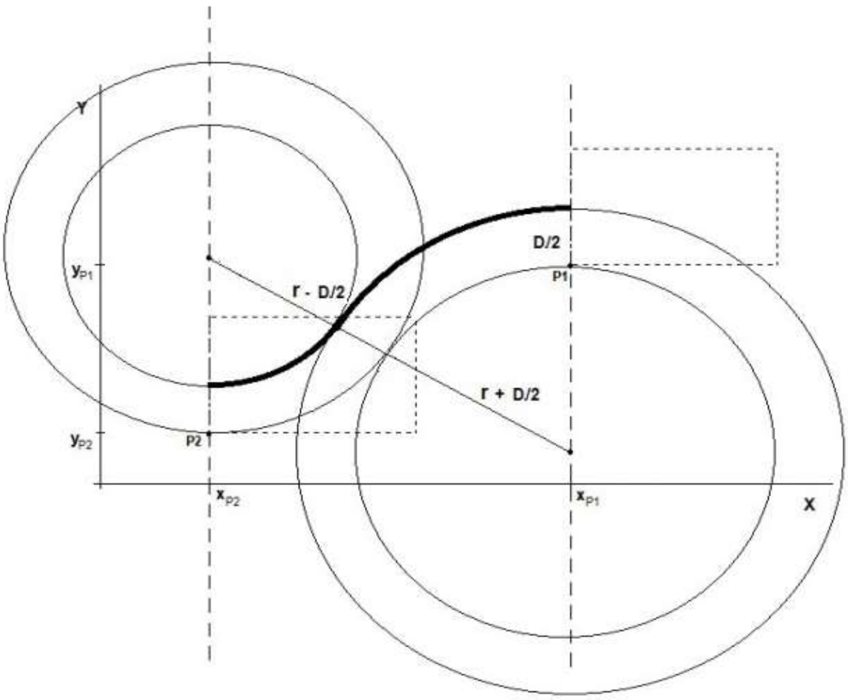
Source: Own Author

In order to design the two circular arcs, and ensure an adequate modeling of the vehicle trajectory planning it is necessary to take into account the triangle $\boldsymbol{E_{init}\ C_r\ C_l}$, according to Al-Kashi's theorem, which is considered an extension of the Pythagorean theorem (STAŃDO et al., 2012) for the triangle $\boldsymbol{E_{init}\ C_r\ C_l}$ (VOROBIEVA et al., 2015). As represented in Equation 4.

$$R_{E_{init}} = \frac{\left(d_{C_l E_{init}}{}^2 - R_{E_{lmin}}{}^2\right)}{2\, R_{E_{lmin}} + 2\, d_{C_l E_{init}}\ \cos\left(\alpha\right)} \tag{4}$$

Figure 3 illustrates the geometric modeling for the parallel parking maneuver.

Figure 3 – Geometric modeling for the parking maneuver



Source: Adapted from VÓRIA (2010)

To define the circular radius in the initial trajectory, the mathematical modeling is based on the Ackerman angle geometry (Θ) applied to the low velocity curves (DIN et al., 2019), determined by the Equation 5.

$$\Theta = arc\ \ tg\left(\frac{L}{R}\right) \quad (5)$$

The orientation (α) of the position initial vehicle when it stops, performs of changing the front wheel angle determined by the following equation 6.

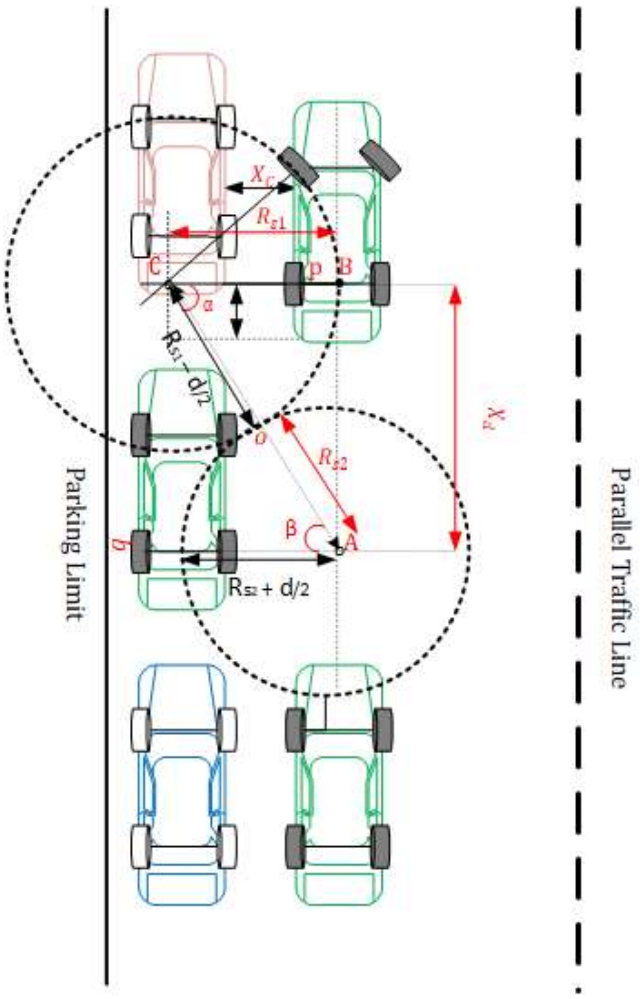
$$\alpha = arc\ sen\ \frac{XP1 - XP2}{2\ r} \quad (6)$$

The width between the center of the rear axle and the wheel to the right of the same axle is described as $\left(\frac{D}{2}\right)$, where (D) defines the width of the vehicle, (r) being the radius of the two circles calculated using the Pythagorean Theorem (GHOSAL, 2021), applied to the right-angled triangle as a whole. However, the radius (r) is determined from Equation 7.

$$r\ = \frac{(YP2 - YP1)^2 + (XP1 - XP2)^2}{4\ (YP1 - YP2)} \quad (7)$$

Figure 4 shows the planning of the parallel parking path and trajectory proposed and validated by computer simulations (MANUEL et al, 2020), considering the main vehicle (green design).

Figure 4 – Path and trajectory of parallel parking



Source: Adapted from MANUEL et al (2020)

For the calculation of R_{s1} (first radius of curvature of the steering angle), it is determined by Equation 8. Where, (d) is the distance between the rear axle and (L) is the distance from the front axle to the rear.

$$R_{s1} = \frac{L}{tan(\theta_{i1})} + \frac{d}{2} \quad (8)$$

The value of R_{s1} varies according to the dimensions of the vehicle from a fixed value of steering angle (θ_{i1}). And for R_{s2} it is calculated in a similar way, according to Equation 9.

$$R_{s2} = \frac{L}{tan(\theta_{i2})} + \frac{d}{2} \quad (9)$$

Where, R_{s2} is defined as the second radius of curvature in relation to the steering angle (θ_{i2}).

The difference between the front inner and external steering angle is calculated by Equation 10.

$$Cot\theta_i - Cot\theta_o = \frac{d}{L} \quad (10)$$

For the parallel automatic parking system (RAZINKOVA et al., 2012) to meet the required maneuver, it is essential to establish minimum parameters regarding the width and length of the parking space.

3. Final considerations

After defining the proper geometric path planning to carry out the parking maneuver, the automatic parking algorithms in parallel when programmed by a microcontroller (or telemetry resources); vehicle trajectory calculation is based on the length and depth (width) information of the space that can be parked. It is assumed that the final position of the rear axle of the vehicle coincides with the distance determined by the length of the vacancy. To ensure that the doors on both sides of the vehicle can open normally, the system sets the width of the target parking space to be at least 70 to 80cm greater than the width of the vehicle body. When a small parking space is detected by ultrasonic sensors, the system recognizes that the parking space is not adequate. The proper planning of the geometric path makes it possible significant solutions as a benefit the reduction of human effort and greater safety during maneuvers.

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